Metal Whiskers:

Failure Modes and Mitigation Strategies

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http://nepp.nasa.gov/whisker

Outline

- A Brief History of Metal Whiskers
- System Failure Modes Caused by Metal Whiskers
- A Few Mitigation Strategies to Reduce Harm From Metal Whiskers

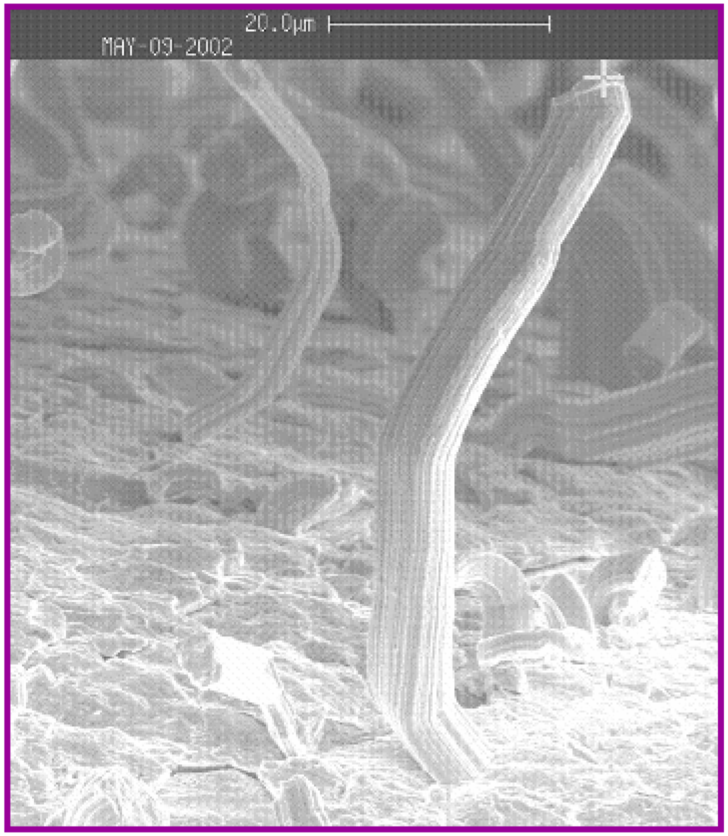
Zinc Whiskers on
Hot Dip Galvanized Steel Pipe

Cover Photo:
Tin whiskers on Tin-Plated Beryllium Copper PCB Card Rails

- *NO WHISKER GROWTH THEORY TO BE DISCUSSED!!!*

What are Tin or Zinc or Cadmium Whiskers?

- **Hair-like structures made of a single grain, or only a few grains, that sometimes erupt from a metal. Coatings of Tin, Zinc and Cadmium are especially able to develop whiskers; but, whiskers have been seen on Gold, Silver, Lead, and other metals too**
- **Growth occurs over time by accretion of metal ions at the base NOT the tip**
- **LENGTH:** **Log-normally distributed**
Rarely up to 10 mm or more
(Typically ~1mm or less)
- **THICKNESS:** **Range 0.006 to >10 um**
(Typical ~ 1 um)
- **Fundamental theories for growth mechanism DO NOT enable prediction of the time-dependence of whisker density, whisker lengths or thicknesses**
  - **To be useful a theory should identify what we must control to make confident predictions**
  - **Such a theory has remained elusive**

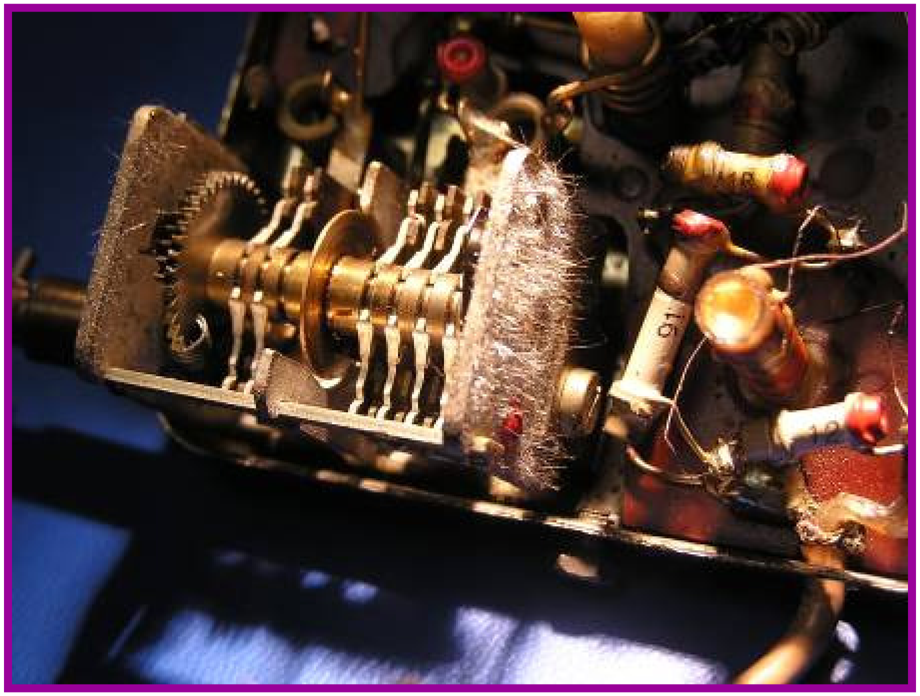


Tin Whiskers on Tin-Plated Ceramic Chip Capacitor

Metal Whiskers
"The Early Years"

- **1946:**
  H. Cobb (Aircraft Radio Corp.) publishes earliest "known" account of CADMIUM whiskers inducing electrical shorting between plates of air capacitors used in military equipment. These events occurred during World War II (~1942 – 1943)

- **1952:**
  Since Cadmium coatings resulted in shorting, Tin and Zinc were used instead. But then K.G. Compton, A. Mendizza, and S.M. Arnold (Bell Labs) reported shorting caused by whiskers from these coatings too!



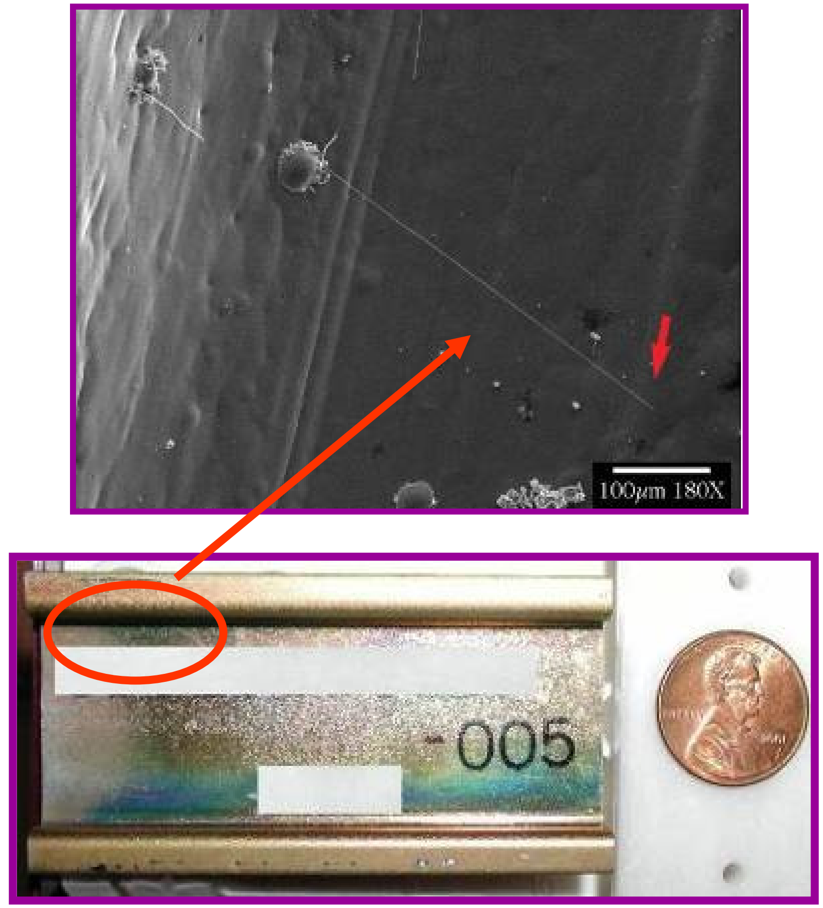
Tin Whiskers on 1960's Era Variable Air Capacitor

Whisker Resistant Metal Coatings
“The Quest”

- **1950s and 60’s:**
  Bell Labs worked through the periodic table, to determine whether addition of some element to a Tin coating would “quench” whiskering
  - ***Adding 0.5 - 1% (by weight) of lead (Pb) works***
  - ***Some additives seem to enhance whiskering***

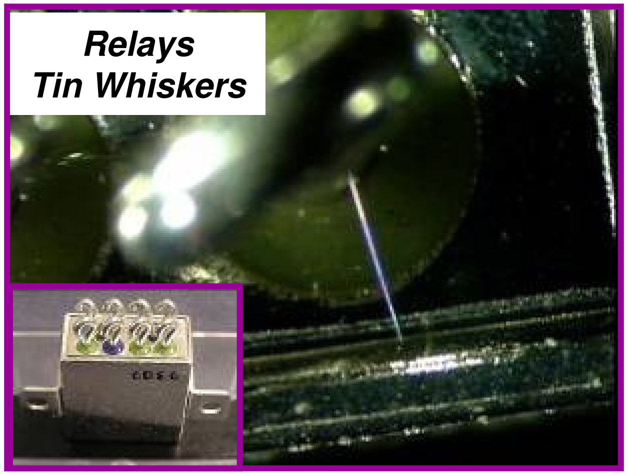
- **Since 1990s:**
  Most MIL specs require adding Pb to any tin coatings used around electronics.
  - **Concentration is usually named as 2% to 3% Pb by weight for “margin”**

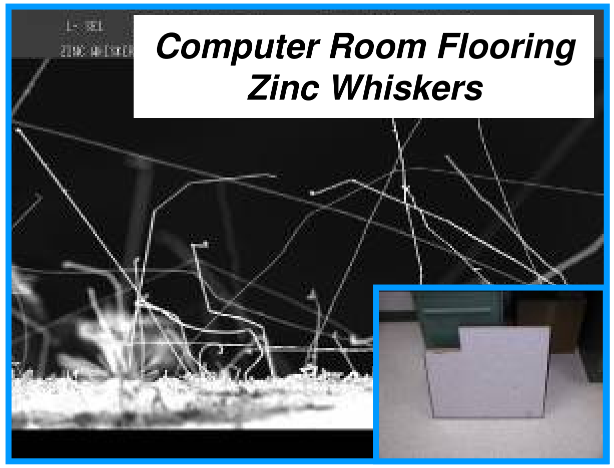
- **What additives quench zinc and cadmium whiskers?**
  - **We don’t know, but certainly NOT chromate conversion finishes!**

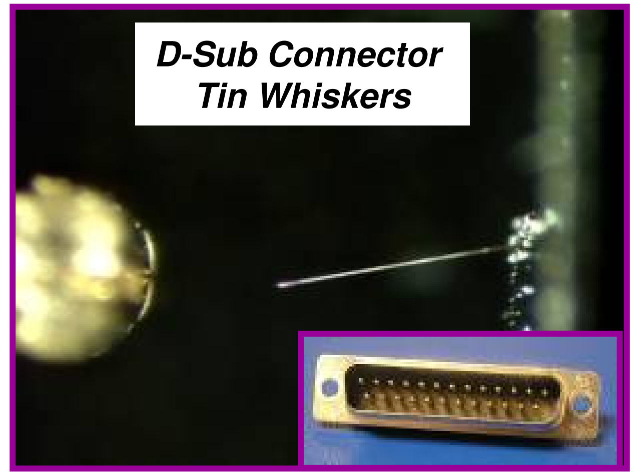


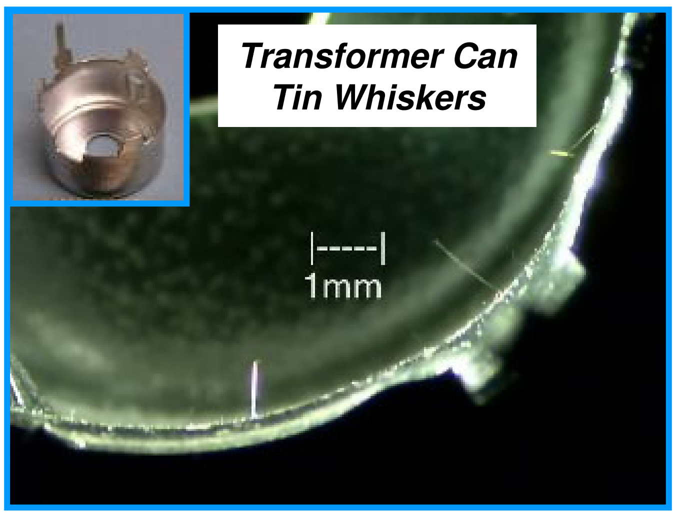
Zinc Whiskers Growing from Zinc-Plated Yellow Chromate Steel Bus Rail

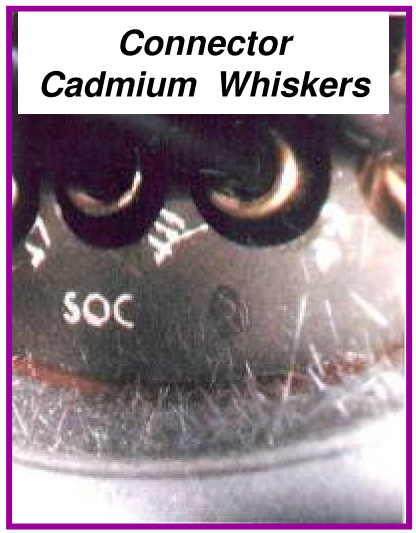
Metal Whiskers on Components

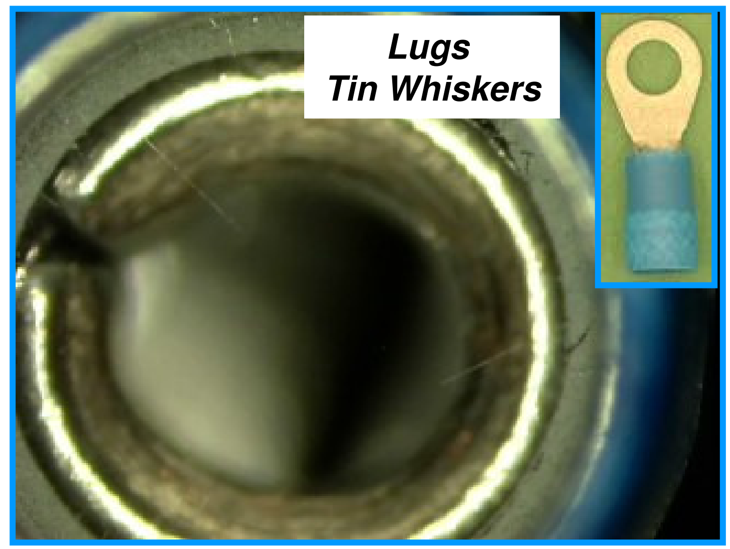






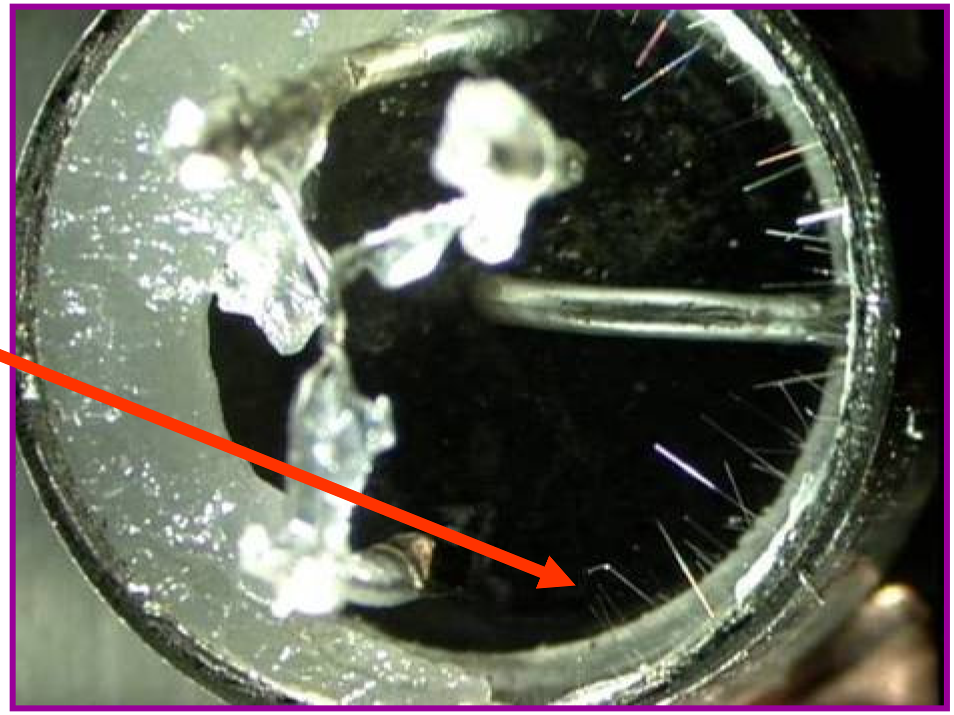






Guess What's Lurking Inside?

1960's Vintage Transistor



Transistor Package is Tin-Plated Inside.

Many Vintage Radio Malfunctions Have Been Attributed to Whiskers Shunting Case to Terminals

http://www.vintage-radio.net/forum/showthread.php?t=5058

2006- NASA GSFC Presented

A Partial History of Documented Metal Whisker Problems

http://nepp.nasa.gov/whisker/reference/tech_papers/2006-Leidecker-Tin-Whisker-Failures.pdf

Year**	Application	Industry	Failure Cause	Whiskers on?
1946	Military	Military	Cadmium Whiskers	Capacitor plates
1948	Telecom Equipme			
1954	Telecom Equipme			
1959	Telecom Equipme			
1959	Telecom Equipme			
1959	Telecom Equipme			
1959	Telecom Equipme			
1986	F15 Radar			
1986	Heart Pacemaker			
1986	Phoenix Missile			
1987	Dresden nuclear Station			
1987	MIL/Aerospace P			
1988	Missile Program "			

Year**	Application	Industry	Failure Cause	Whiskers on?
1990	Apnea Monitors	Medical (RECALL)	Zinc Whiskers	Rotary Switch
1990	Duane Arnold Nuclea Power Station			
1992	Missile Program "C"			
1993	Govt. Electronics			
1995	Telecom Equipment			
1996	Computer Routers			
1996	MIL Aerospace			
1998	Aerospace Electroni			
1998	Computer Hardware			
1998	DBS-1 (Side 1)			
1998	Dresden nuclear Pov Station			
1998	GALAXY IV (Side 2)			
1998	GALAXY VII (Side 1)			
1998	Military Aerospace			
1998	PAS-4 (Side 1)			
1999	Eng Computer Cente			
1999	SOLIDARIDAD I (Side			
1999	South Texas Nuclear			
199X	Telecom Equipment			

Year**	Application	Industry	Failure Cause	Whiskers on?
2000	GALAXY VII (Side 2)	Space (Complete Loss)	Tin Whiskers	Relays
2000	Missile Program "D"	Military	Tin Whiskers	Terminals
2000	Power Mgmt Modules	Industrial	Tin Whiskers	Connectors
2000	SOLIDARIDAD I (Side 2)	Space (Complete Loss)	Tin Whiskers	Relays
2001	GALAXY IIIR (Side 1)	Space	Tin Whiskers	Relays
2001	Hi-Rel	Hi-Rel	Tin Whiskers	Ceramic Chip Caps
2001	Nuclear Power Plant	Power	Tin Whiskers	Relays
2001	Space Ground Test Eqpt	Ground Support	Zinc Whiskers	Bus Rail
2002	DirecTV 3 (Side 1)	Space	Tin Whiskers	Relays
2002	Electric Power Plant	Power	Tin Whiskers	Microcircuit Leads
2002	GPS Receiver	Aeronautical	Tin Whiskers	RF Enclosure
2002	MIL Aerospace	MIL Aerospace	Tin Whiskers	Mounting Hardware (nuts)
2002	Military Aircraft	Military	Tin Whiskers	Relays
2002	Nuclear Power Plant	Power	Tin Whiskers	Potentiometer
2003	Commercial Electronics	Telecom	Tin Whiskers	RF Enclosure
2003	Missile Program "E"	Military	Tin Whiskers	Connectors
2003	Missile Program "F"	Military	Tin Whiskers	Relays
2003	Telecom Equipment	Telecom	Tin Whiskers	Ckt Breaker
2004	Military	Military	Tin Whiskers	Waveguide
2005	Communications	Radio (1960s vintage)	Tin Whiskers	Transitor TO Package
2005	Millstone Nuclear Power Plant	Power	Tin Whiskers	Diode (Axial Leads)

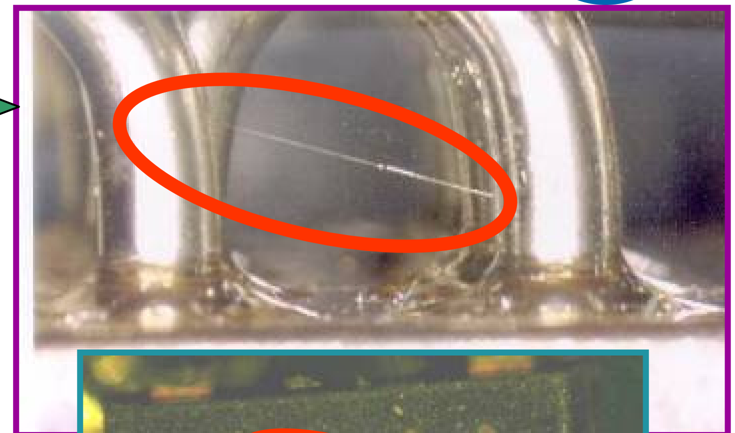
These are ~10% of the Problems We Know About

"There is a name for those who suppose that doing the same thing will produce different results. That name is 'Idiot'."

- Albert Einstein

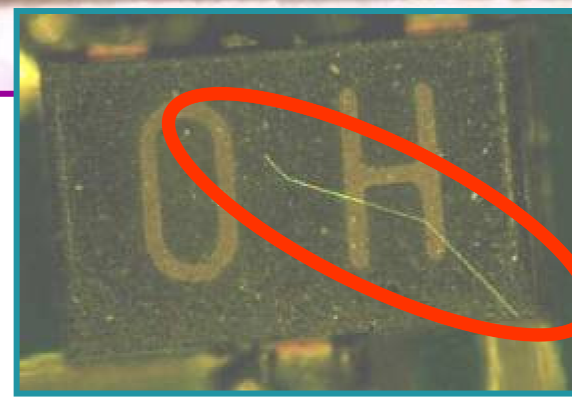
Basic Whisker Failure Modes

Electrical Short Circuits



- Continuous short if $I_{whisker} < I_{melt}$
- Intermittent short if $I_{whisker} > I_{melt}$

Debris/Contamination



- Interfere with Sensitive Optics or MEMS
- Produce Shorts in Areas REMOTE From Whisker Origins
  (Zinc Whiskers on raised flooring are a PRIME Example)

METAL VAPOR ARC

- If $I_{whisker} >> I_{melt}$ Whisker Can Vaporize into a Metal Gas
- If EMF is sufficiently high, then the metal gas can be ionized into a conductive PLASMA of Metal Ions
- **Plasma Can Form an Arc Capable of Carrying** ***HUNDREDS OF AMPS!*** Depends on arc gap length, voltage, current, pressure, etc.

Metal Whisker Melting Current -- Pt. 1

$$I_{melt,vac} = \left[\frac{2\sqrt{LzT_0}}{R_0}\right]\cos^{-1}\left(\frac{T_0}{T_{melt}}\right)$$

See Backup Slides for Derivation

– ***Where*** ***$Lz \sim 2.45 \times 10^{-8}\ (V/K)^2$ is the Lorenz number, T_{melt} = melting temperature, T_0 = ambient temperature, R_0 = whisker resistance at ambient***

Material	***T_{melt}***	***$I_{melt, vac}$***	***$V_{melt} = R_0 * I_{melt, vac}$***
Tin	505.1K	87.5 mV / R_0	88 mV
Cadmium	594.2K	97.1 mV / R_0	97 mV
Zinc	692.7K	104.4 mV / R_0	104 mV

If $V_{whisker} > V_{melt}$
Then the Whisker will Fuse Open

But there is MORE to this story

Metal Whisker Melting Current -- Pt. 2

- Metal whiskers are also coated with electrically insulating films
  - Tin oxides, Zinc oxides, moisture films, et al

- Direct MECHANICAL contact by the whisker to another conductor does NOT guarantee ELECTRICAL contact
  - For Electrical Contact, the potential difference must exceed "dielectric breakdown" of the insulating films
  - For tin and zinc whiskers, independent groups have confirmed the oxide film breakdown ranges from ~ 0.2V to ~ 45V

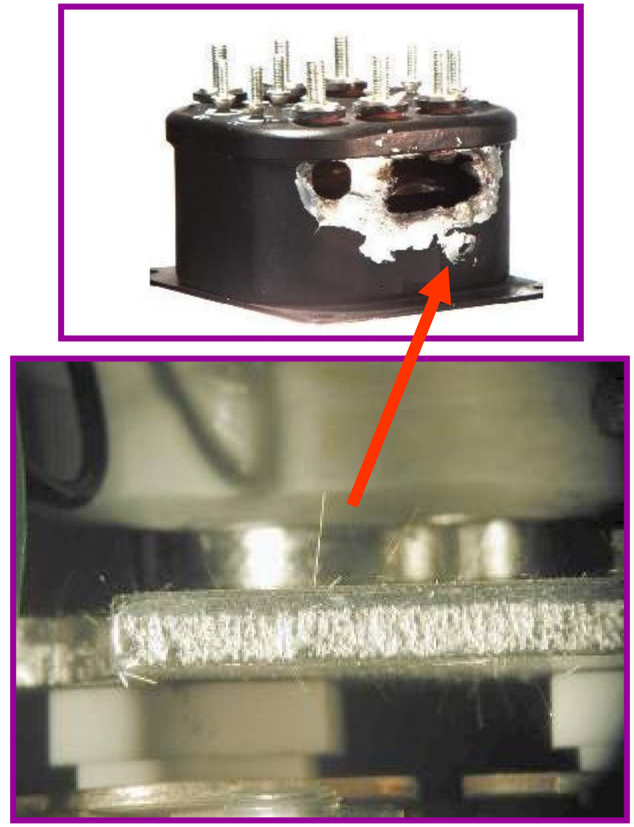
Electrical Current Flow through this Tin Whisker did not occur until $V_{applied}$ ~ 3 V at which point the Whisker promptly melted

See Backup Slides for a Suggested Circuit for Measuring Metal Whisker Resistance

Sustained Metal Vapor Arcing

- Gaps shorter than a few tens of microns can support a sustained arc at potential differences of 12 to 14 volts, and currents of 0.1 to 0.3 amperes.
  – See “Electrical Contacts” by Paul G. Slade, part three

- However, as the arc gap increases, the EMF needed to sustain the arc increases, as does the current.
  – GSFC testing of FM08 style fuses with metal filaments ~5 mm long finds ~ 75 volts at more than 30 amperes is needed to generate a sustained arc across this arc gap when P ~1 torr



Tin Whiskers Growing on Armature Of Relay Produced Metal Vapor Arc

G. Davy, "Relay Failure Caused by Tin Whiskers", Northrop Grumman, Technical Article, October 2002
http://nepp.nasa.gov/whisker/reference/tech_papers/davy2002-relay-failure-caused-by-tin-whiskers.pdf

How do People with "Whiskers" Cope?

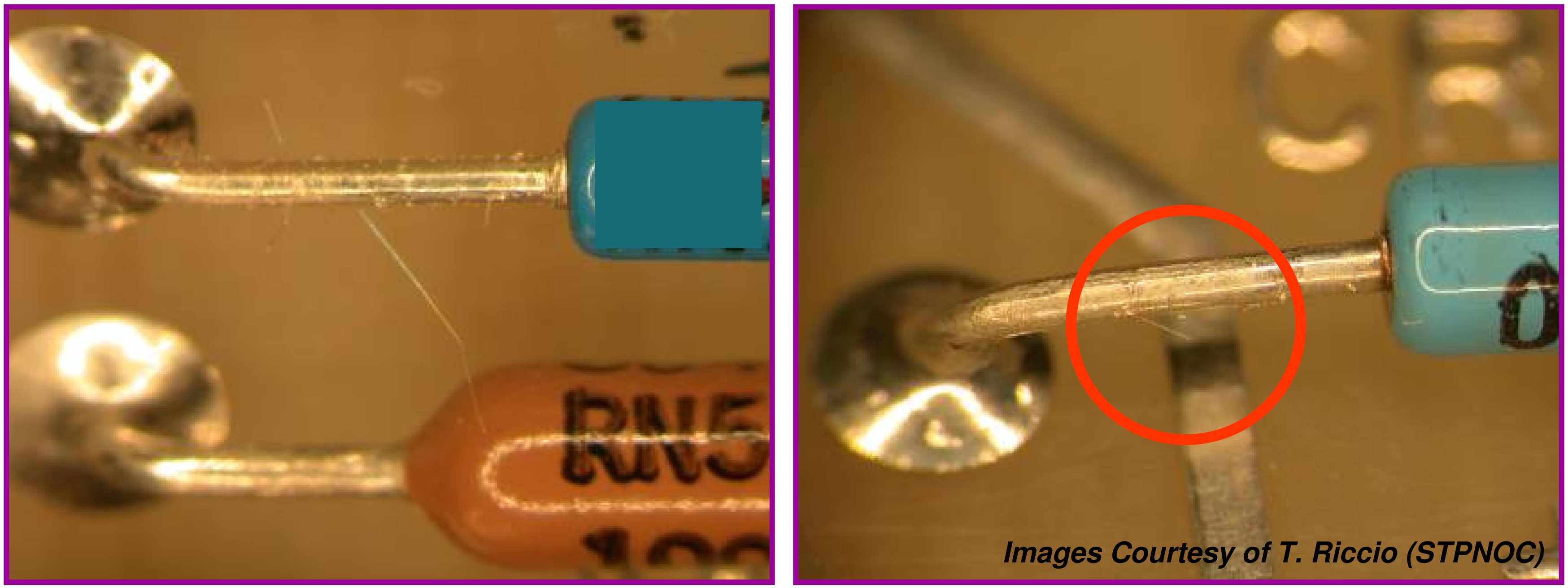
My Whisker ***"Stress Relaxation Theory"***

Man with "Facial Whiskers"
Does YOGA!

Men with "Metal Whiskers"
Find Innovative Ways to Relieve Stress

A Case for Whisker Mitigation Strategies?

Tin Whiskers on Tin-Plated Axial Leaded Diodes



Images Courtesy of T. Riccio (STPNOC)

- PWB and components were **NOT Conformal Coated**
- Diode Leads were **NOT Hot Solder Dipped**

Three Whisker Mitigation Strategies

*Mitigation – to make **less** severe or painful*
Merriam-Webster Dictionary

Risk "Mitigation" ≠ Risk "Elimination"

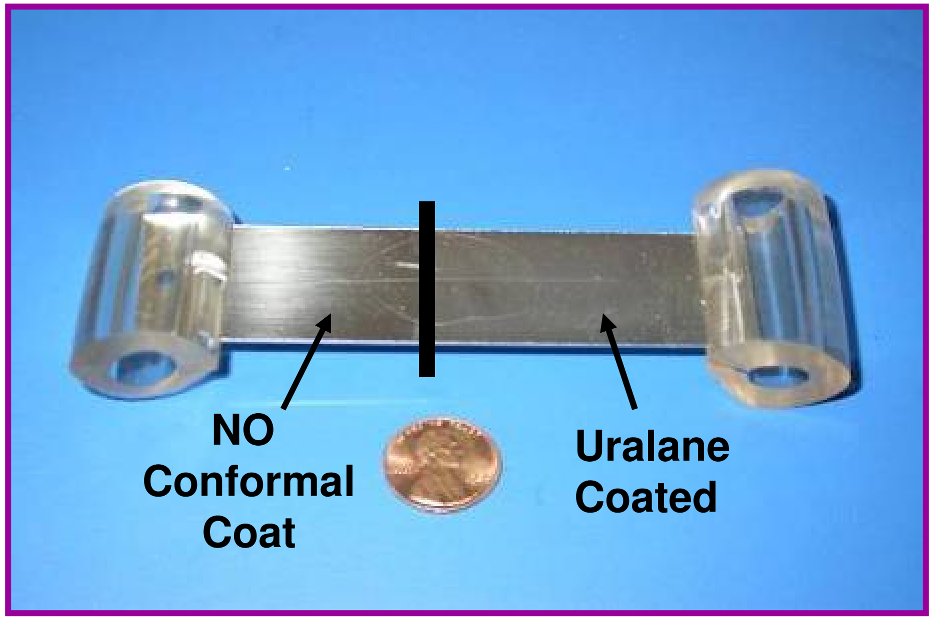
- Avoid Use of Whisker Prone Surface Finishes
  - Perform independent materials composition analysis
  - *"Trust, But VERIFY!"* using X-ray Fluorescence (XRF), Energy Dispersive X-ray Spectroscopy (EDS), et al
- Conformal Coat
  - Can slow whisker growth
  - Can block whiskers from electrically shunting distant conductors
- Remove/Replace Tin Finishes When Practical
  - Hot Solder Dip using lead-tin (Pb-Sn) solders
  - "First Do No Harm" Principle

NASA Goddard Whisker Mitigation Study
Conformal Coat (Uralane 5750* Polyurethane)
~9 Years of Office Ambient Storage

- **Specimen:**
  - 1” x 4”x 1/16” Brass 260
  - Tin-Plated 200 microinches
  - Intentional scratches created after plating

- **Conformal Coating:**
  - Uralane 5750 on ½ of sample
  - Nominal Thickness = 2 mils

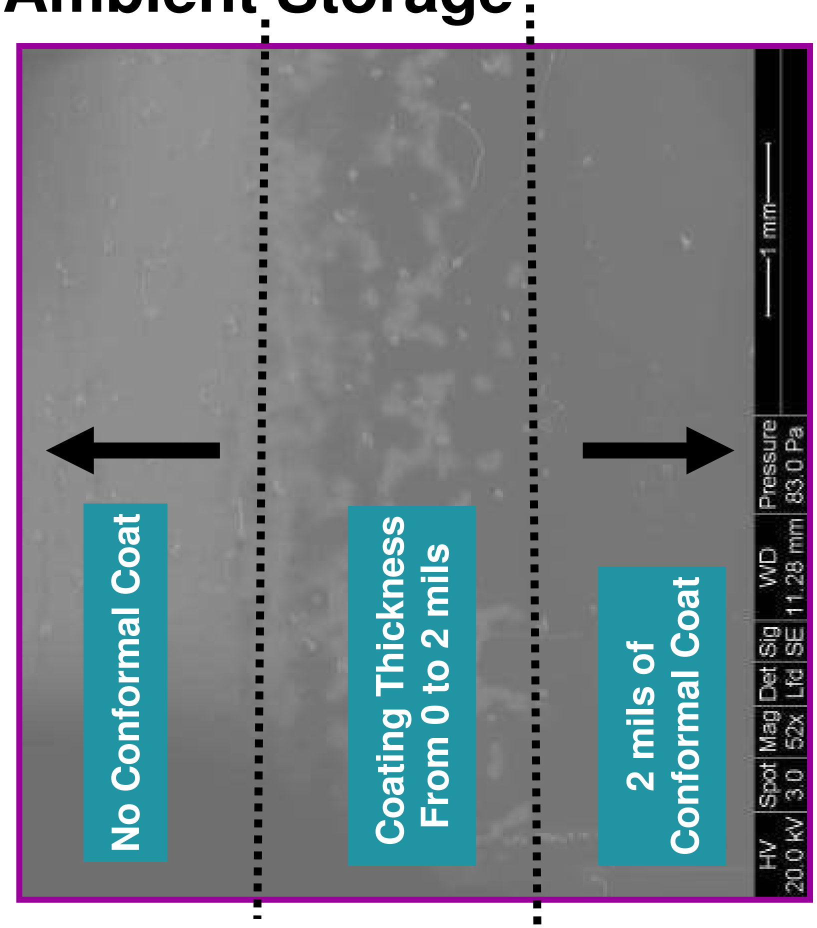
- **Storage Conditions:**
  - Office Ambient ~ 9 years



***** Uralane™ 5750 now known as Aralane™ 5750*

NASA Goddard Whisker Mitigation Study
Conformal Coat (Uralane 5750* Polyurethane)
~9 Years of Office Ambient Storage

- **Coating Thickness Can Vary Depending on Process Parameters**
- **NASA Test Coupons had a "transition" region ~2 mm wide where the conformal coating thickness was variable between 0 and 2 mils**
- ***One must understand their own processes to ensure the coating thickness is sufficient everywhere you intend it to be!!***

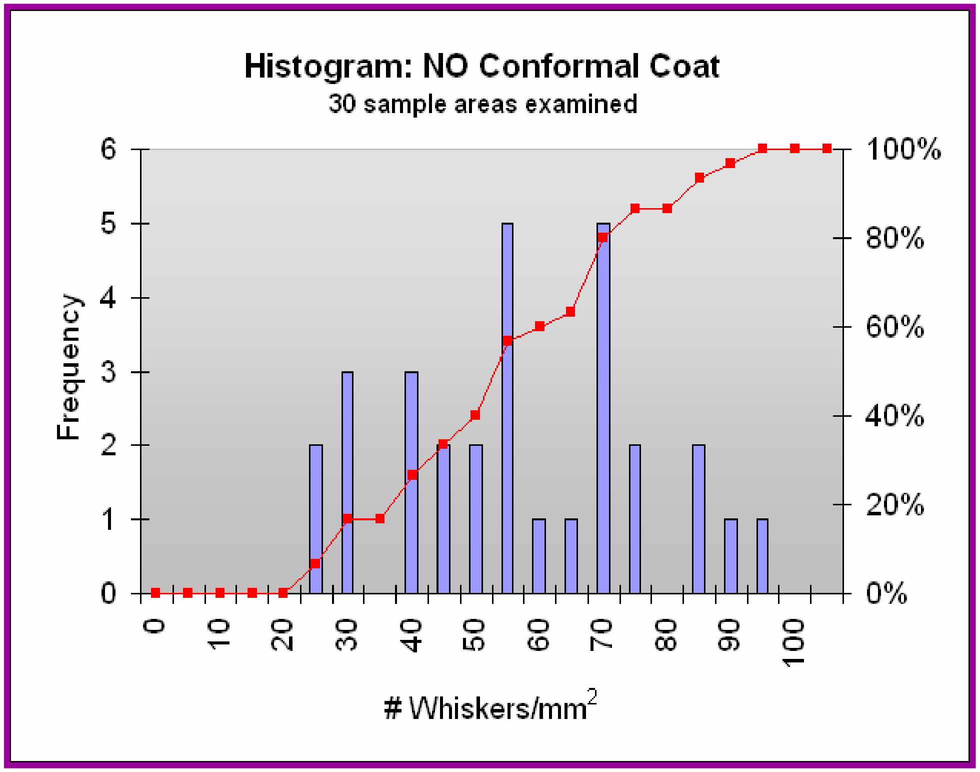


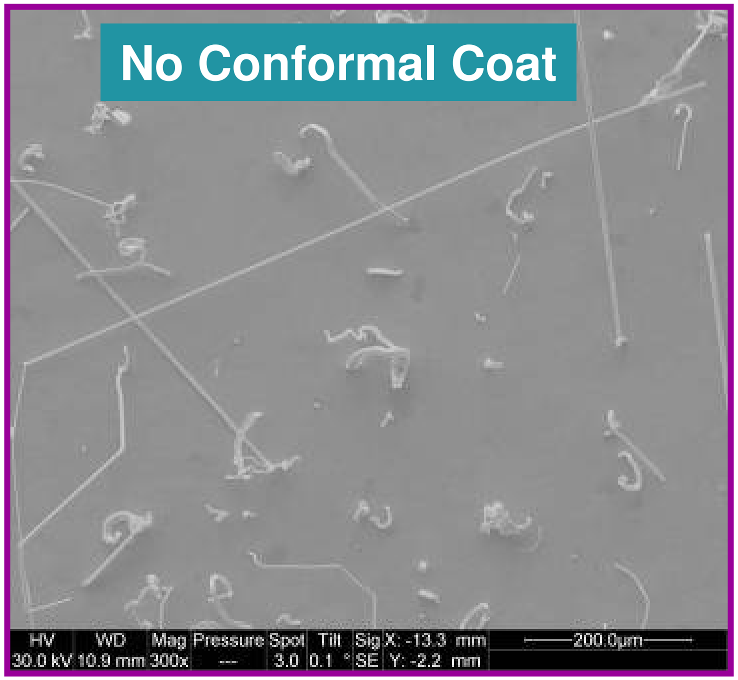
NASA GSFC Conformal Coat Tin Whisker Test Coupon

** Uralane™ 5750 now known as Aralane™ 5750*

Control Areas – No Conformal Coat
9-Years of Office Ambient Storage

- Control Areas Grew Whiskers Abundantly
  - Avg: 55 ± 19.6 whiskers / mm^2
  - Range: 23 to 95 whiskers / mm^2

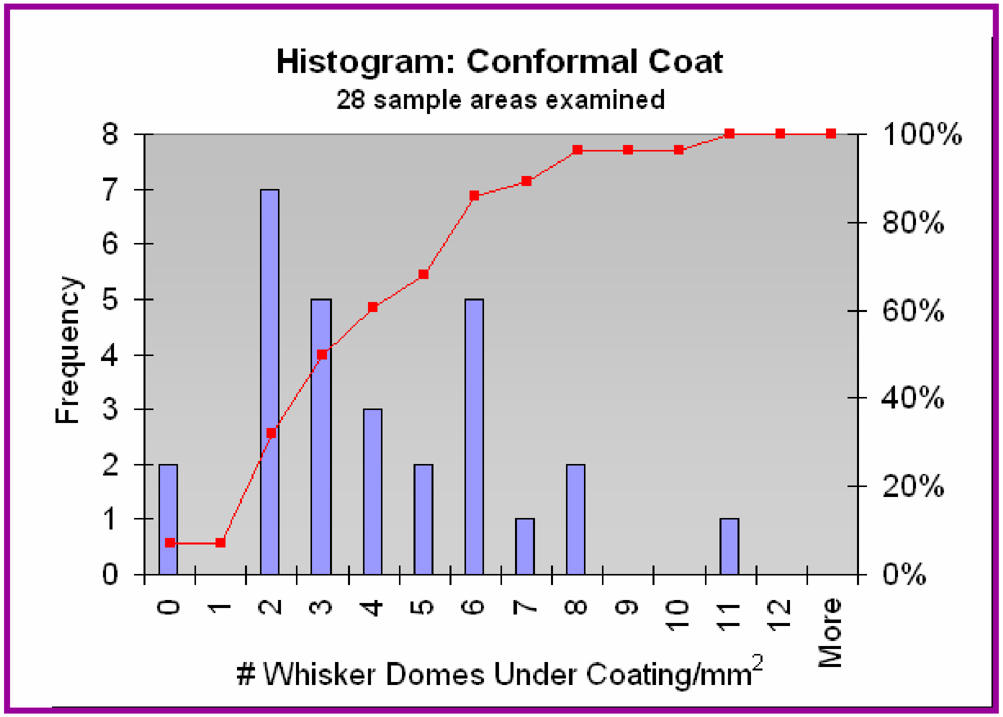


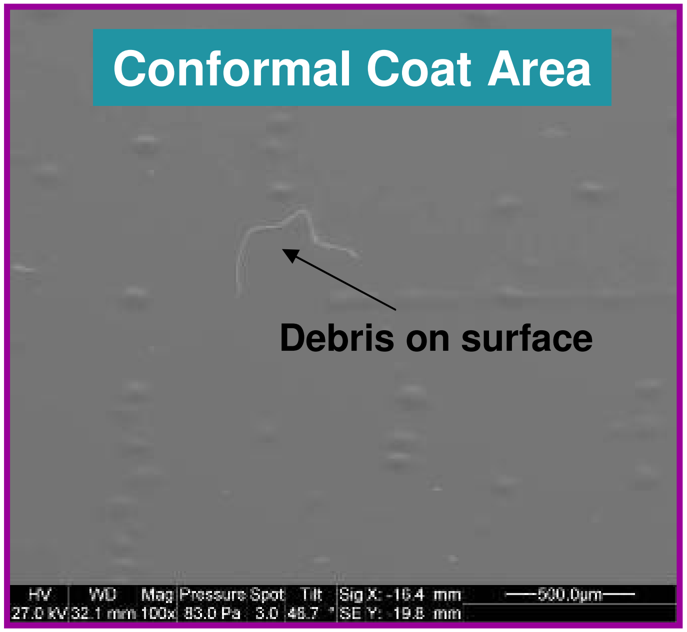


Uralane 5750 – 2 Mils Thick
9-Years of Office Ambient Storage

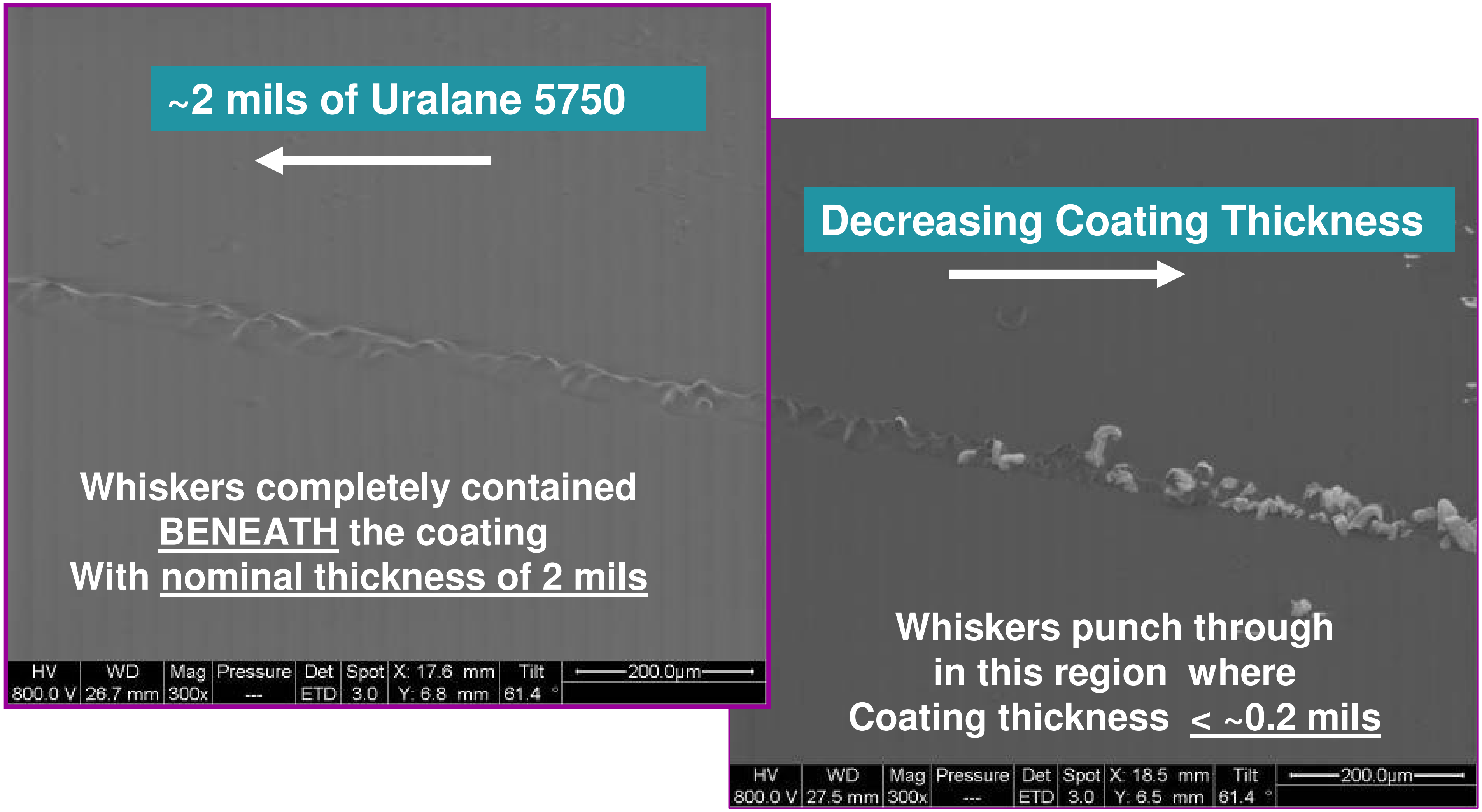
- **Conformal Coated Areas Grew Whiskers Too**
  - ***To date ALL whiskers are contained beneath the coating that is <u>2 mils thick</u>***
  - ***With SEM we cannot see into the coating. Instead we see "domes" caused by whiskers that lift coating slightly***
  - **Avg: 3.4 ± 2.6 domes / mm^2**
  - **Range: 0 to 10.6 domes / mm^2**

We suspect we are only counting "thick" whiskers in this statistic because the "thin" ones mechanically buckle before they can lift the coating enough to produce visible "domes"



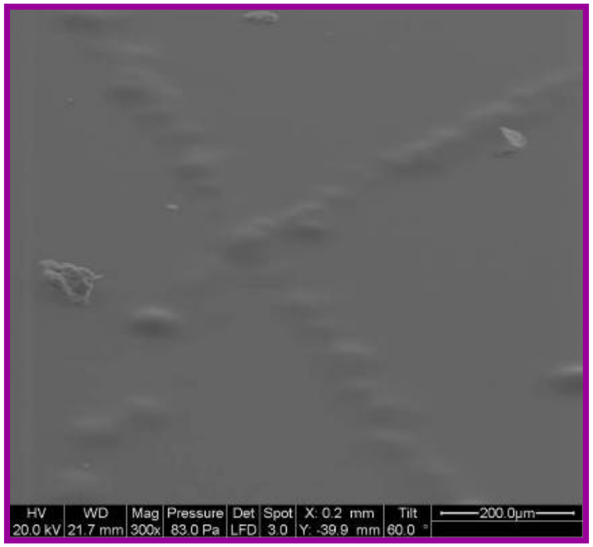


Whisker Puncture vs. Coating Thickness



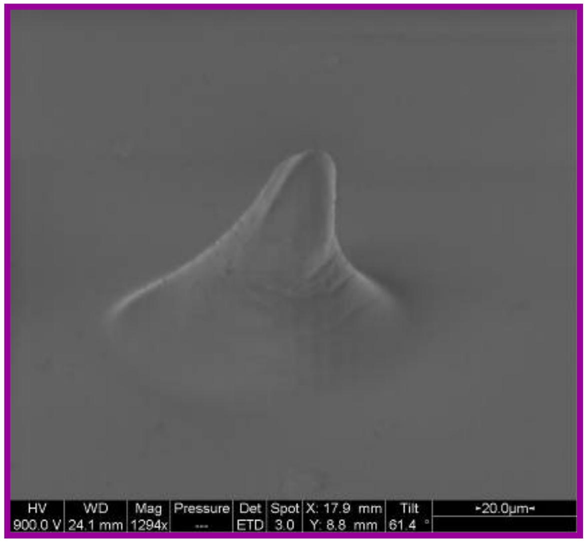
Uralane 5750 Conformal Coat - 9-Years of Office Ambient Storage

2 Mils Uralane = Very Effective



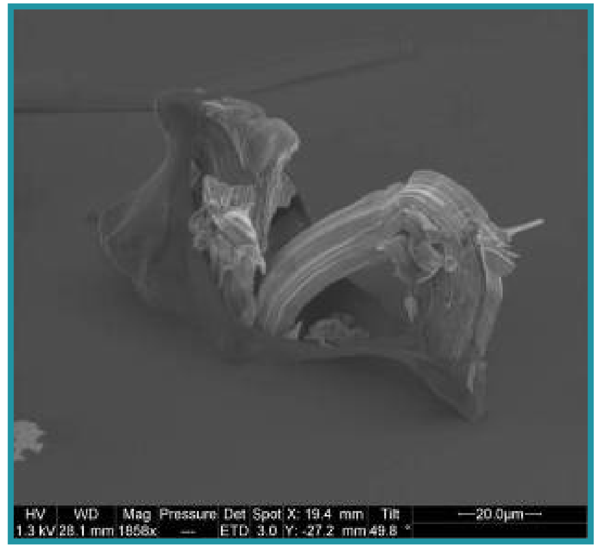
Whiskers Completely Entrapped Under the Coating → Euler Buckling

~0.5 Mils Uralane = Less Effective



Whisker "Lifting" Coating into Shape of Circus Tent, But Not Yet Penetrating

~0.1 Mils Uralane = Not Effective

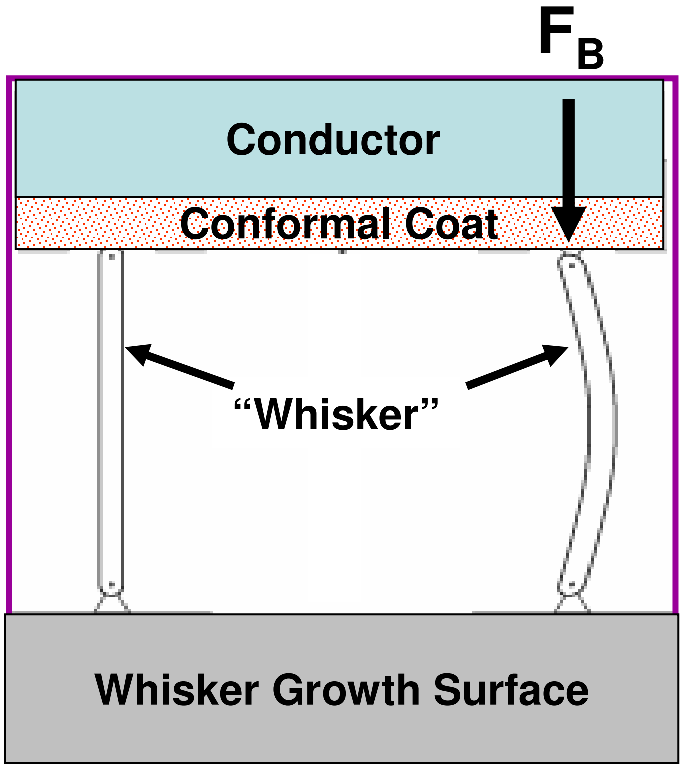


Whiskers Breaking Through "Thin" Coating

Euler Buckling
Axial Force Required to Buckle a Metal Whisker

$$F_B = \frac{\pi^2 EI}{(KL)^2} \approx \left(\frac{\pi^3 \cdot E}{32}\right)\left(\frac{d^4}{L^2}\right)$$

E = Young's Modulus of whisker material,
I = Area Moment of Inertia,
(e.g. $I = \pi d^4 / 64$ for circular cross section)
L = Length of whisker,
K = Column Effective Length Factor
K = 0.5 for whisker fixed at both ends
K = 0.7 for fixed at one end, pinned at other

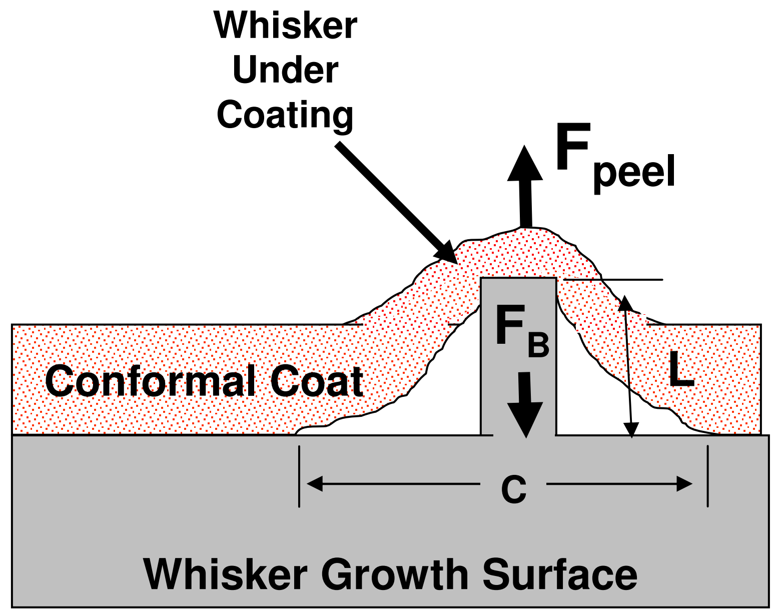


Whiskers Lift and Peel Conformal Coat Until Whisker Buckles OR Coating Fails (F_{peel} vs. F_{Buckle})

- As whisker first emerges it is short and stiff thus $F_B > F_{peel}$ and whisker begins to lift the coating forming a "circus tent" with height L = length of whisker;

- "Tent" joins the surface at a circle of circumference C ~ 2πQL,
  - Q describes the details of tent-like shape

- To peel conformal coating up and away from the surface, one needs to apply a force (F_{peel}) proportional to the circumference:
  - $F_{peel} = \Phi * C = 2 \pi Q \Phi L$
    Φ = peel strength of material which describes the adhesion of the coating to the tin, and the effect of the separation angle. It also depends on the rate at which the coating is peeled away.



Uralane 5750 has better self-cohesion than adhesion to a tin surface

Additional Analysis Pending

Will Whiskers Buckle Before Puncturing a Distant Coated Surface?

- The displacement of the conformal coat due to a whisker pushing against the coating is:

$$D = \left(\frac{1-\nu^2}{E_{coat}}\right)\left(\frac{F_B}{d}\right) \approx \left(\frac{\pi^3}{32}\right)\left(1-\nu^2\right)\left(\frac{E_W}{E_{coat}}\right)\left(\frac{d^3}{L^2}\right)$$

Where

D = Displacement of conformal coat

ν = Poisson's ratio

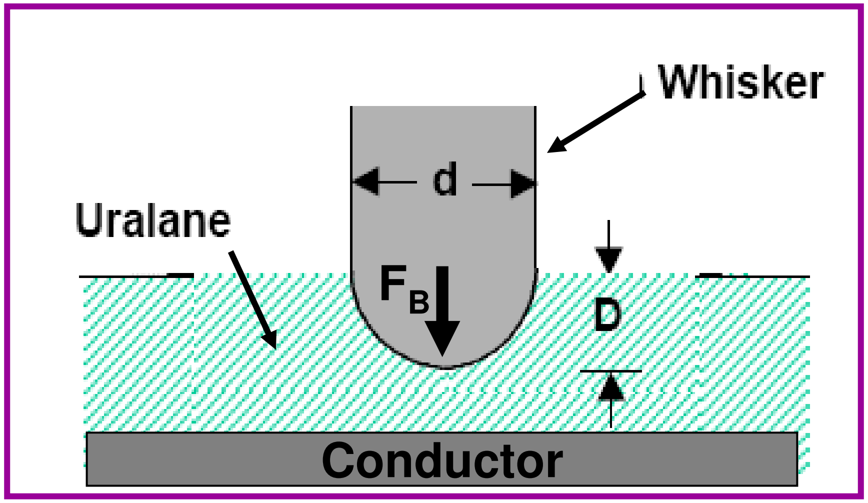
E_{coat} = Young's Modulus of coating

E_W = Young's Modulus of Whisker

d = "Diameter" of whisker

L = Length of whisker

F_B = Euler Buckling Strength of the whisker



Thank Goodness for Euler Buckling and Conformal Coat on this PWB!!!

Photo Credit: M&P Failure Analysis Laboratory
The Boeing Company Logistics Depot

Effects of Conformal Coating -- 1

- Numerous sorts of coatings have been tried:
  - Reports of success vary from “none” to “perfect”, sometimes for the same sort of coating.

- NASA GSFC has used Uralane 5750, applied to pre-primed tin-plated surfaces to a thickness of 2 mils (=50 micrometers) +/- 10%:
  - After ~9 years of office ambient storage, these surfaces have whiskered abundantly, but the number of whiskers escaping through the 2 mil thick areas has been zero

- Dr. Thomas Woodrow (Boeing) has studied Urethane (acrylic) coatings, a silicone coating, and Parylene C coating of varying thicknesses up to ~ 4 mils (= 100 micrometers):
  - Some whiskers have penetrated even the thickest coatings when exposed to 25°C / 97% R.H.

Effects of Conformal Coating -- 2

- Conclusion 1:
  - Uralane 5750, applied to at least 2 mils thickness, is a substantial improvement over an uncoated surface.

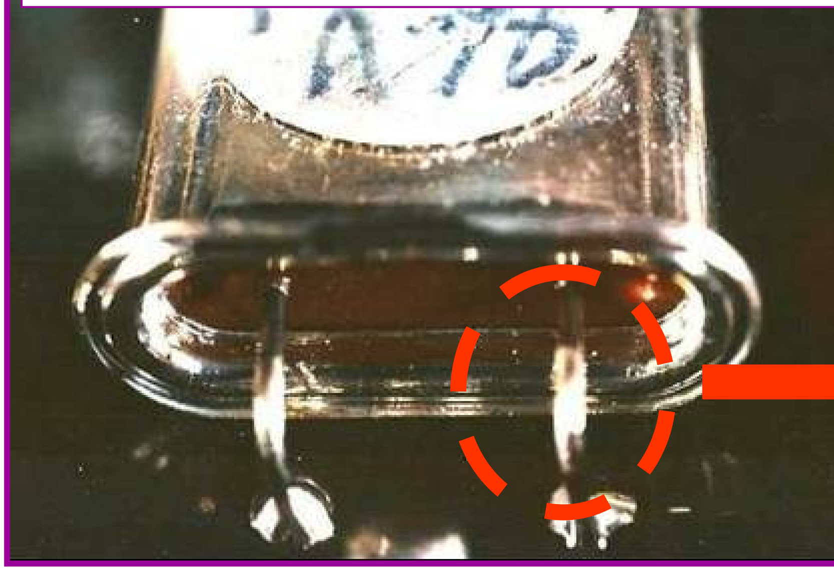
- Conclusion 2:
  - It is possible to suppose the surface is protected when it is not
  - There can be “weak zones” of thin coating allowing vertical escape

- Conclusion 3:
  - Even “poor” coatings can offer some protection against a whisker coming from a distant source and attempting to contact the protected surface --- long whiskers bend easily (Euler Buckling).
  - Conformal coat protects against a conductive bridge from detached whiskers lying across a pair of conductors

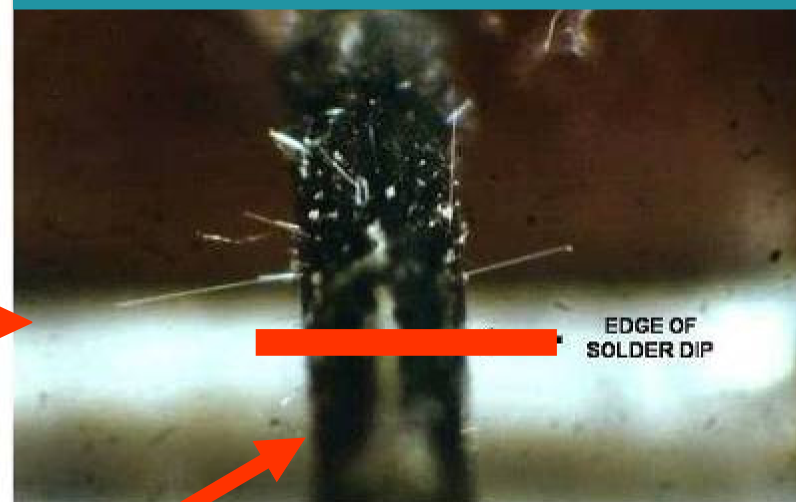
Hot Solder Dip
Benefits & Limitations

Field Failure ONE Year **After Assembly**

Crystal with Tin-Plated Kovar Leads (with Nickel Underplate)



Tin Whiskers (~60 mils) Grew on *NON-Dipped* Region Shorting to Case Causing Crystal to Malfunction



- Leads were *Hot Solder Dipped* (Sn63Pb37) *within 50 mils* of Glass Seal BEFORE Mounting to enhance solderability
- Dip was not 100% of leads due to concerns of inducing harm to glass seal

- No Whiskers on Hot Solder Dipped Surface
- ABUNDANT whiskers on the Non-Dipped Surface

Contact Information

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Work Performed in Support of the
NASA Electronic Parts and Packaging (NEPP) Program

Acknowledgment to Dr. Michael Osterman
University of MD – Center for Advanced Life Cycle Engineering (CALCE)

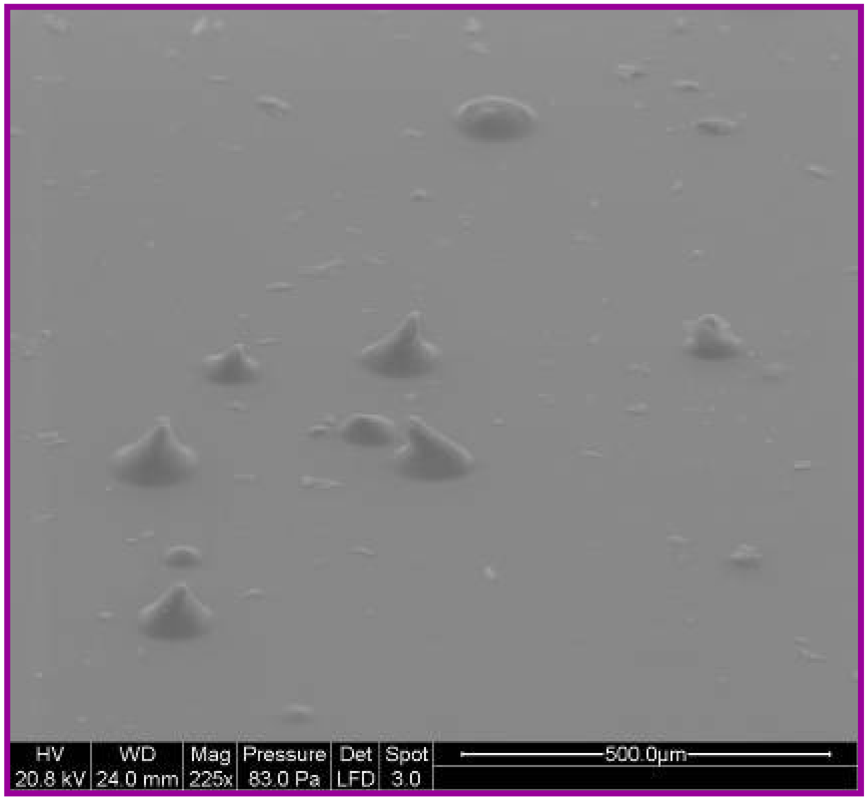
NASA Tin and Other Metal Whisker WWW Site

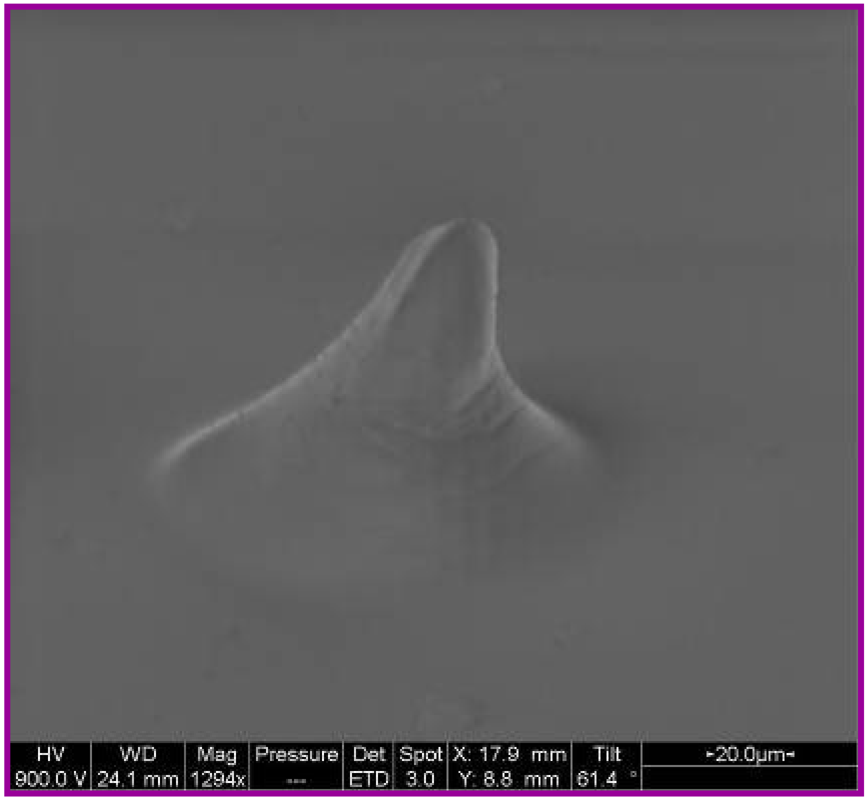
http://nepp.nasa.gov/whisker

Backup Slides

Tin Whiskers Forming "Circus Tents" in Thin Uralane 5750 Conformal Coat - 9-Years of Office Ambient Storage

Coating Thickness < 0.5 Mil

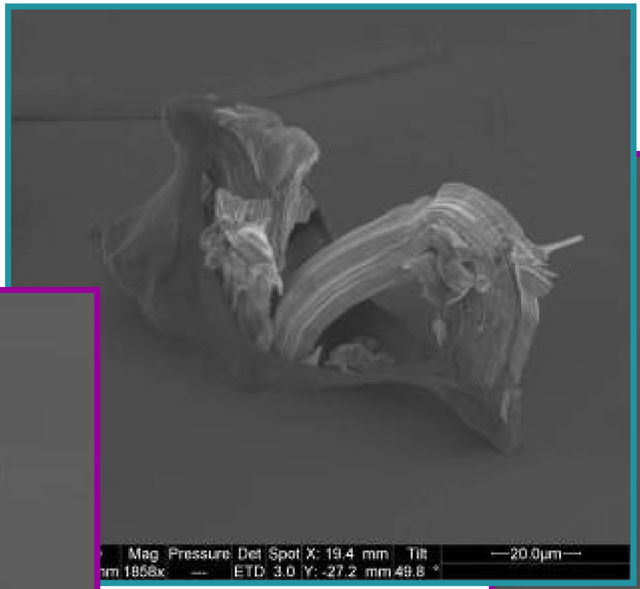


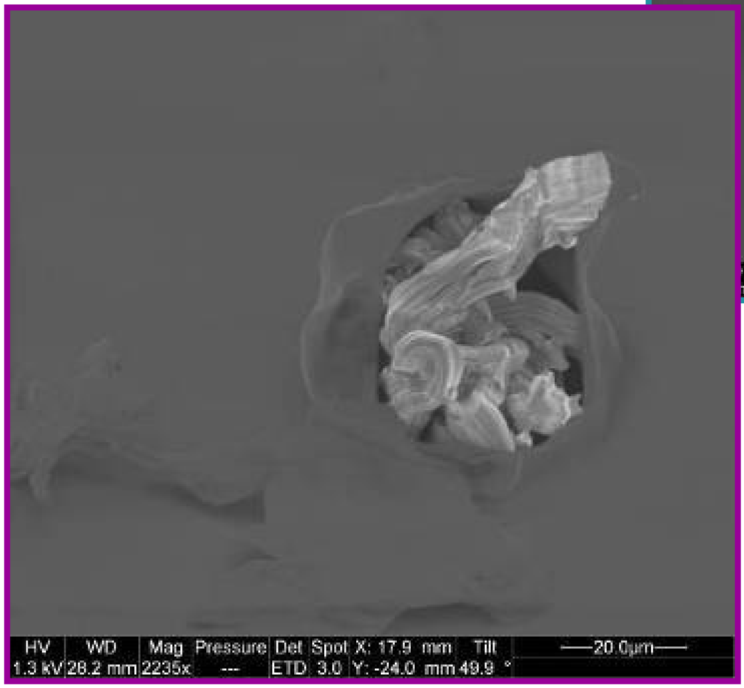


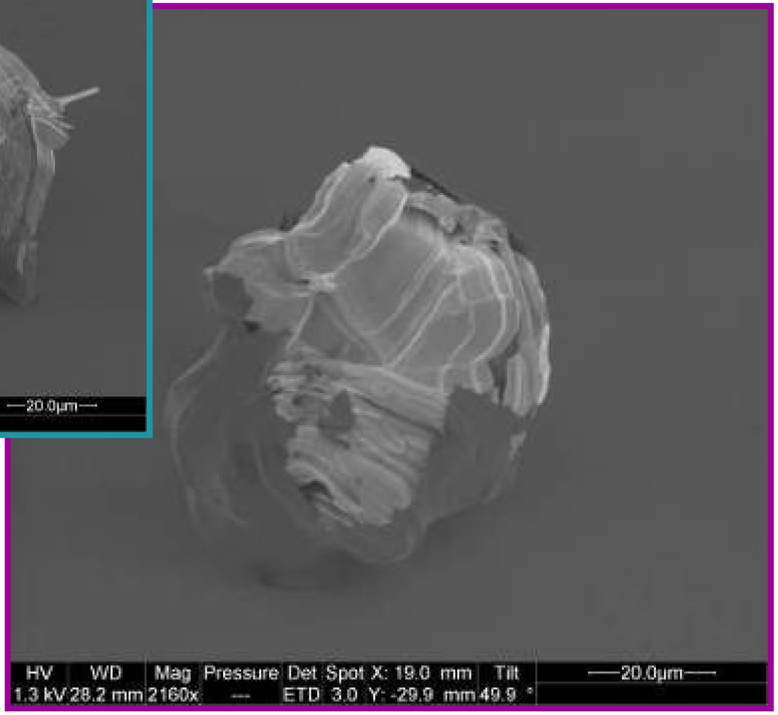
Tin Whiskers Rupturing THIN Coating

~0.1 to 0.2 Mils Uralane 5750 Conformal Coat

9-Years of Office Ambient Storage







A Few Recent Whisker Experiences

- **Tin Whiskers:**
  - **2005:** Connecticut Nuclear Power Plant shutdown due to Tin whiskers on diodes
  - **2006:** Space Shuttle Transportation System discovers Tin whiskers on card rails: Some 100 to 300 million whiskers were in OV-105's boxes
  - **2006:** SWATCH reports 30% of crystals made using RoHS-compliant tin-copper solder sprouting Tin whiskers. 5% catastrophically shorted within months.

- **Zinc Whiskers:**
  - **2005:** Colorado State Government builds a new “disaster recovery center” after Zinc whiskers crippled its old data center
  - **2005**: 75% of the computer equipment in a particular data center failed due to Zinc whiskers from raised floor tiles. Root cause identified in ~8 months later
  - **2006:** Persistent NAVY weapon system failure confirmed caused by Zinc whiskers

- **Cadmium Whiskers:**
  - **2006**: Spaceflight program observes Cadmium whiskers on electrical switch
  - **2007**: Test chamber feed-thru failure caused by Cadmium whiskers on connector shells

Circuit to Measure Resistance of a Metal Whisker

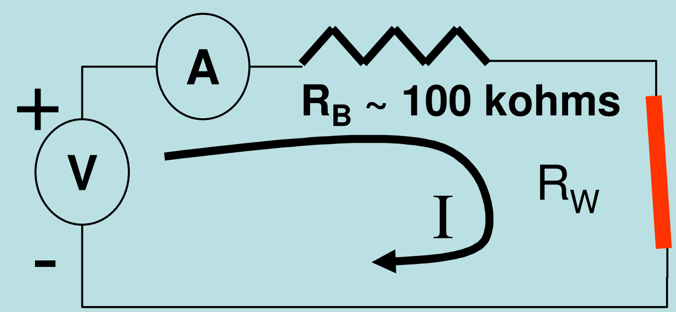
- **Use of a simple "Ohmmeter" to measure the resistance of a metal whisker is NOT preferred**
  - **Ohmmeter may supply $V_{out} < V_{breakdown}$ for the insulating films (oxides, moisture) that form on a metal whisker**
  - **Ohmmeter may supply $V_{out} > V_{melt}$ causing the whisker to melt before resistance can be measured**

- **Instead, a variable power supply and a ballast resistor should be used to overcome the above complications**
  - **Adjust $V_{out} > V_{breakdown}$ of insulating films on whisker**
  - **When $V_{out} > V_{breakdown}$, R_B quickly drops $V_{whisker} < V_{melt}$**

Circuit to measure R_w

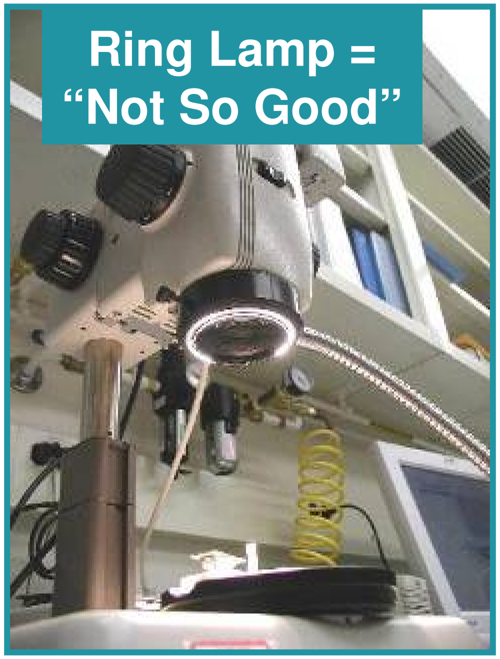
$$\boxed{R_W = \frac{V - IR_B}{I}}$$

Choose R_B such that $V_{whisker} < 80\% \ V_{melt}$



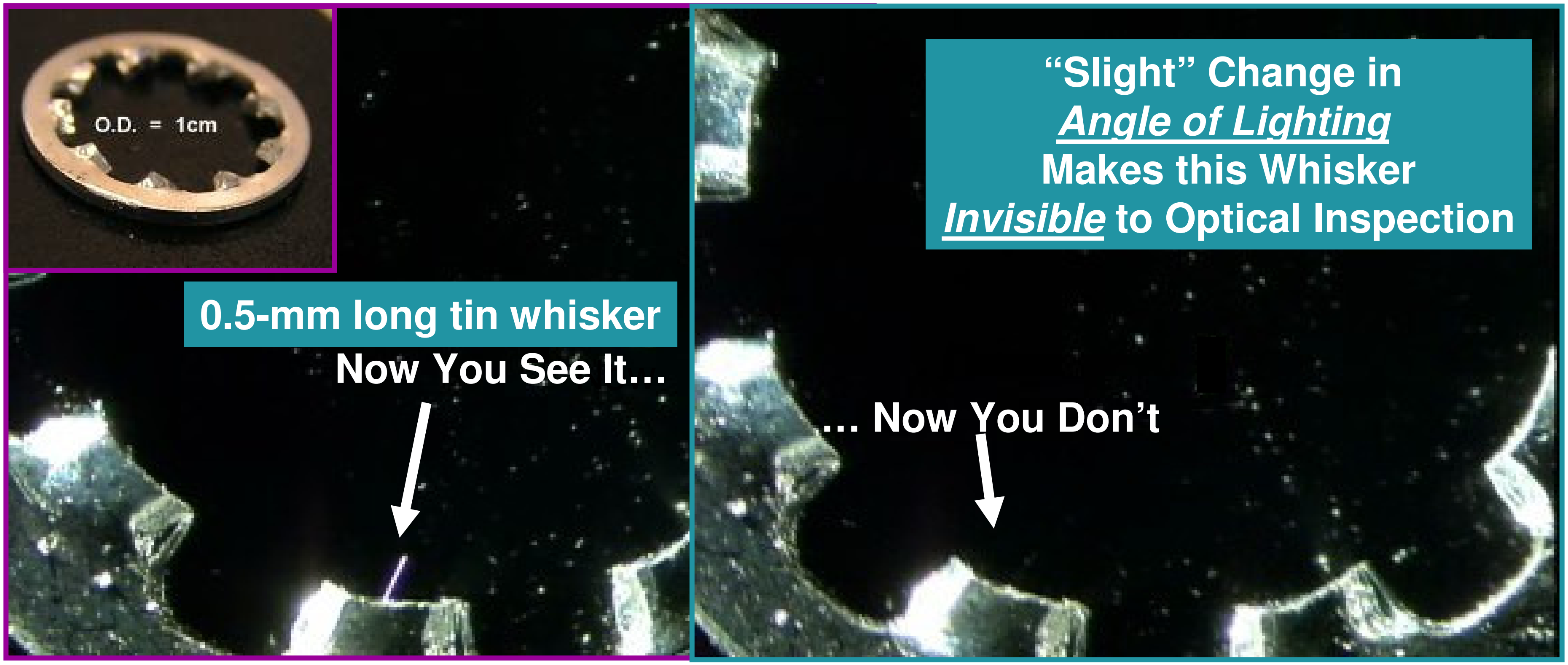
Optical Inspection for Metal Whiskers

- Basic Equipment:
  - Binocular Microscope
  - Light Source: Flex Lighting PREFERRED over Ring Lamp
- Freedom to tilt sample and/or lighting to illuminate whisker facets is VERY IMPORTANT



Evidence of "Absence of Whiskers"? (Optical Microscopy)

Tin-Plated Lock Washer

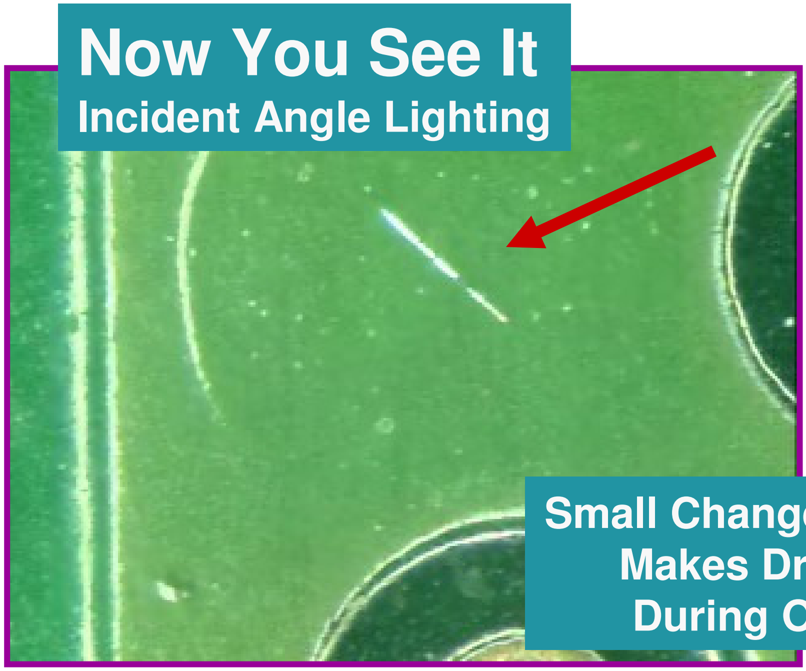


The absence of evidence is NOT evidence of absence

Field Technicians and Failure Analysts Need To Be Acquainted with Metal Whiskers!!!

NASA GSFC has published videos to aid in optical inspection for metal whiskers

http://nepp.nasa.gov/whisker/video



Small Change in Angle of Lighting Makes Dramatic Difference During Optical Inspection

Why Are Tin, Zinc, Cadmium Still Used?

- Not all Tin (or Zinc or Cadmium) surfaces whisker!
  - Rough estimate: 3% to 30% do whisker.
- Not all metal whiskers cause shorts
  - Environment (geometry and electrical potentials matter).
  - Rough estimate: 3% to 30% do short.
- Not all whisker-induced shorts are traced to whiskers
  - They are very hard to see and failure analysis techniques often destroy evidence
  - Rough estimate: 0% to 10% are correctly traced.
- Not all identified whisker adventures are reported
  - Rough estimate: 0% to 3% are reported, once identified
- Hence, we expect between 0.00% and 0.03% of shorting problems caused by these coatings to be reported
  - While some 0.1% to 10% of these coatings are actually causing shorts.
  - With such a few public cases, many say “What, me worry?”
- Whiskering is dramatically inhibited when 0.5% (or more) lead (Pb) is added to Tin coatings: the shorting rate then approaches zero
  - This has been the case for the Hi-Rel community
  - But Pb use is being restricted by international legislation, and so the shorting rate may jump to 10% from zero **==> SWATCH GROUP <==**

"The Five Stages of Metal Whisker Grief"

By Henning Leidecker

Adapted from Elisabeth Kubler-Ross in her book "On Death and Dying", Macmillan Publishing Company, 1969

Denial

"Metal whiskers?!? We ain't got no stinkin' whiskers! I don't even think metal whiskers exist! I KNOW we don't have any!"

Anger

"You say we got whiskers, I rip your $%#@ lungs out! Who put them there --- I'll murderize him! I'll tear him into pieces so small, they'll fit under one of those *^&$#% whiskers!"

Bargaining

"We have metal whiskers? But they are so small. And you have only seen a few of them. How could a few small things possibly be a problem to our power supplies and equipment? These few whiskers should be easy to clean up."

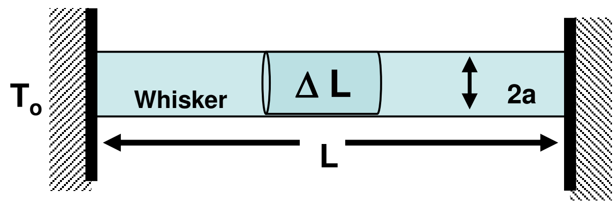
Depression

"Dang. Doomed. Close the shop --- we are out of business. Of all the miserable bit joints in all the world, metal whiskers had to come into mine... I'm retiring from here... Going to open a 'Squat & Gobble' on the Keys. "

Acceptance

"Metal whiskers. How about that? Who knew? Well, clean what you can. Put in the particle filters, and schedule periodic checks of what the debris collectors find. Ensure that all the warrantees and service plans are up to date. On with life."

Derivation of Melting Current of a Metal Whisker in Vacuum



Assume both ends of Whisker are thermally grounded to T_o

$$\frac{du}{dt} + \Phi = source$$

du/dt **+** ***Φ*** **=** ***source***

$$u = C \cdot T \qquad c = \frac{C}{V}$$

$$u = \left(\frac{C}{V}\right) \cdot V \cdot T = c \cdot V \cdot T$$

$$u = c \cdot \Delta L \cdot A \cdot T$$

$$\boxed{\frac{du}{dt} = c \cdot \Delta L \cdot A \cdot \frac{\partial T}{\partial t}}$$

$$\Phi = \left(\frac{\partial J}{\partial x}\right) \cdot \Delta L \cdot A$$

Convection loss = 0 for vacuum
Neglect radiation loss

$$J = -k_T \cdot \frac{\partial T}{\partial x} \qquad \frac{\partial J}{\partial x} = -k_T \cdot \frac{\partial^2 T}{\partial x^2}$$

$$\Phi = -k_T \cdot \left(\frac{\partial^2 T}{\partial x^2}\right) \cdot \Delta L \cdot A \qquad k_T = \frac{L_Z \cdot T}{\rho}$$

$$\boxed{\Phi = -\frac{L_Z T}{\rho}\left(\frac{\partial^2 T}{\partial x^2}\right) \cdot \Delta L \cdot A}$$

$$source = I^2 \cdot R$$

$$I = J_e \cdot A \qquad R = \frac{\rho \cdot \Delta L}{A}$$

$$source = \left(J_e^2 \cdot A^2\right) \cdot \left(\frac{\rho \cdot \Delta L}{A}\right)$$

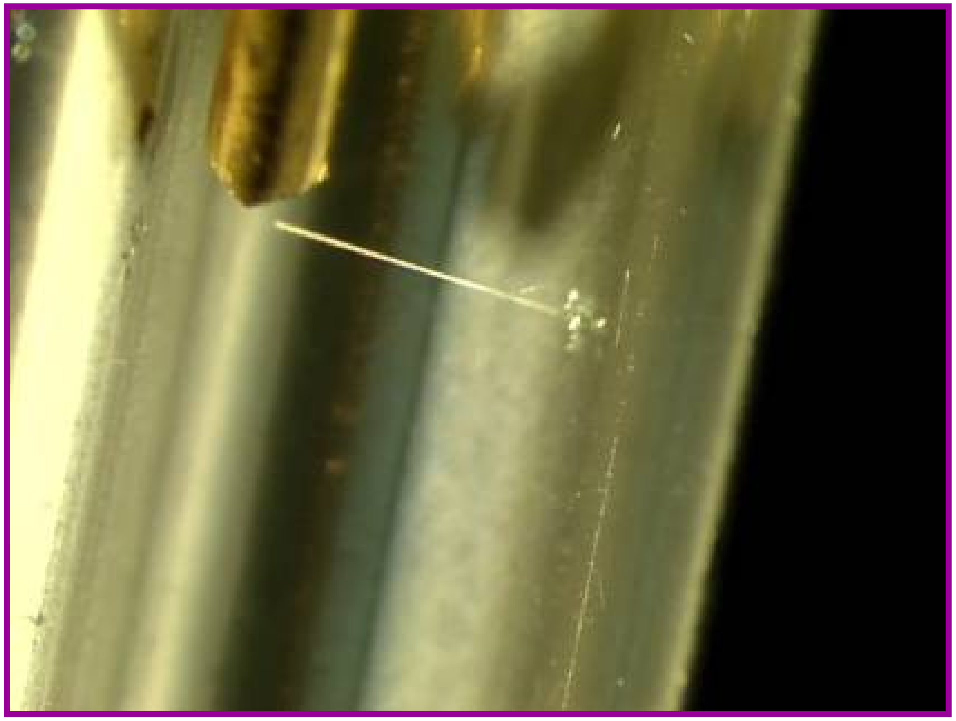
$$\boxed{source = \left(J_e^2 \cdot A\right) \cdot \rho \cdot \Delta L}$$

$$\left[c \cdot \Delta L \cdot A \cdot \frac{\partial T}{\partial t}\right] - \left[\frac{L_z \cdot T}{\rho}\left(\frac{\partial^2 T}{\partial x^2}\right) \cdot \Delta L \cdot A\right] = J^2 \cdot \rho \cdot \Delta L \cdot A$$

$$\left[c \cdot \frac{\partial T}{\partial t}\right] - \left[\frac{L_Z \cdot T}{\rho}\left(\frac{\partial^2 T}{\partial x^2}\right)\right] = J^2 \cdot \rho$$

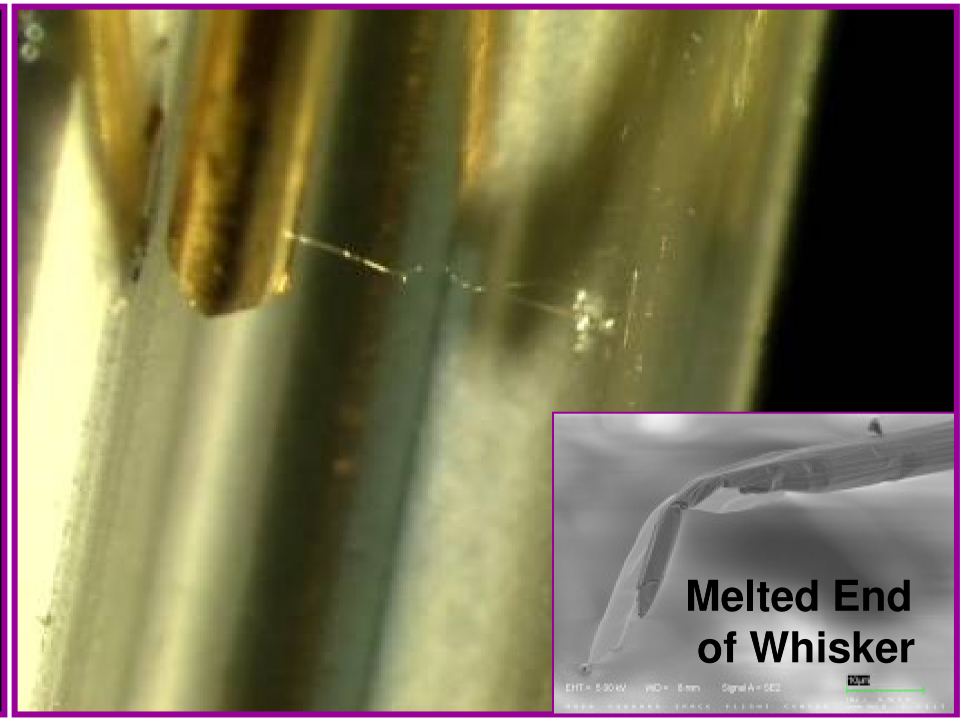
$$\boxed{I_{melt,vac} = \left[\frac{2\sqrt{L_Z T_0}}{R_0}\right] \cos^{-1}\left(\frac{T_0}{T_{melt}}\right)}$$

An Example of “Melting” a Tin Whisker



Before Contact

1. Gold-Plated Test Probe has +3 Volts Relative to Tin Whisker



After Contact

1. Tip of whisker micro-welds to gold test probe
2. Whisker melts mid-length
3. Small section of whisker root remains attached to substrate